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# Sensitivity/Uncertainty Analysis of a Borehole Scenario Comparing Latin Hypercube Sampling and Deterministic Sensitivity Approaches

**Technical Report**

**October 1983**


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## ABSTRACT

A computer code was used to study steady-state flow for a hypothetical borehole scenario. The model consists of three coupled equations with only eight parameters and three dependent variables. This study focused on steady-state flow as the performance measure of interest. Two different approaches to sensitivity/uncertainty analysis were used on this code. One approach, based on Latin Hypercube Sampling (LHS), is a statistical sampling method, whereas, the second approach is based on the deterministic evaluation of sensitivities.

The LHS technique is easy to apply and should work well for codes with a moderate number of parameters. Of deterministic techniques, the direct method is preferred when there are many performance measures of interest and a moderate number of parameters. The adjoint method is recommended when there are a limited number of performance measures and an unlimited number of parameters. This unlimited number of parameters capability can be extremely useful for finite element or finite difference codes with a large number of grid blocks.

The Office of Nuclear Waste Isolation will use the technique most appropriate for an individual situation. For example, the adjoint method may be used to reduce the scope to a size that can be readily handled by a technique such as LHS. Other techniques for sensitivity/uncertainty analysis, e.g., kriging followed by conditional simulation, will be used also.

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# 1 INTRODUCTION

The Office of Nuclear Waste Isolation (ONWI) will conduct extensive sensitivity/uncertainty studies as part of their performance assessment of a high-level nuclear waste repository in salt. The assessments will quantify performance measures and their related uncertainty. The sensitivity of the performance measures will be evaluated using computer models. These models focus primarily on continuous processes such as ground-water flow with discrete events (e.g., human intrusion) superimposed on the process behavior.

The purpose of this report is to provide a description of two sensitivity/uncertainty analysis approaches: (1) a statistical sampling method based on Latin Hypercube Sampling (LHS); and (2) a deterministic evaluation of sensitivities coupled with a parameter covariance matrix for uncertainty. Both the direct and adjoint deterministic methods are described. A hypothetical borehole flow scenario was used to demonstrate these analysis technologies. The types of information obtainable from the various techniques are described. Analysis results are provided and comparisons of results are presented.

## 2 DEMONSTRATION SCENARIO

A hypothetical scenario is described in which a borehole is drilled through an aquifer above a nuclear waste repository, through the repository, to an aquifer below. The flow through this borehole is a function of parameters such as the radius of the borehole, hydraulic potential difference between lower and upper aquifers, conductivity of fill material, transmissivity of upper and lower aquifers, etc. Each parameter varies considerably. For this scenario, eight input parameters are considered and analyses are made using a simple computer code.

Figure 2-1 provides a schematic view of a borehole drilled from the ground surface through a high-level nuclear waste repository and the aquifers above and below the repository horizon. As a conservative assumption, the borehole is considered as screened through the full depths of both aquifers. For this analysis, it is assumed that the potentiometric head is higher in the upper aquifer than in the lower. This results in downward flow. This condition is found in the bedded salt sites being considered for a potential nuclear waste repository.

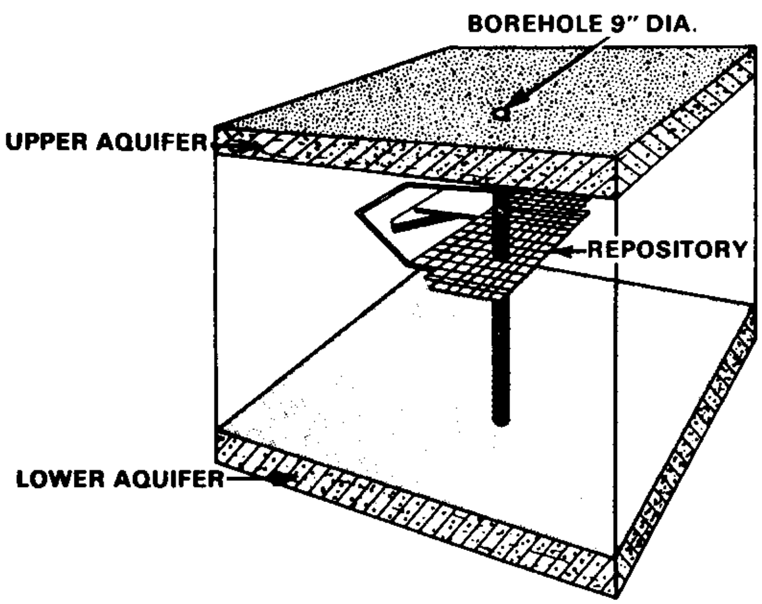


Figure 2-1. Borehole Demonstration Schematic.

Considering a fully penetrating well and no regional ground-water gradient, the steady-state flow through upper aquifer to borehole in an aquifer system is defined by Equation 1:

$$Q = \pi 2 \, T_u \frac{(H_u - H_{wu})}{\ln (r/r_w)} \tag{1}$$

where:

$Q$ = flow, $m^3/yr$

$T_u$ = transmissivity of upper aquifer, $m^2/yr$

$H_u$ = potentiometric head of upper aquifer, m

$H_{uw}$ = steady-state potentiometric head in borehole at upper aquifer, m

$r$ = radius of influence, m

$r_w$ = radius of borehole, m

Using assumptions of Equation 1, the steady-state flow from borehole to lower aquifer can be described by Equation 2:

$$Q = -2\pi T_l \frac{(H_l - H_{wl})}{\ln (r/r_w)} \tag{2}$$

where:

$T_l$ = transmissivity of lower aquifer, $m^2/yr$

$H_l$ = potentiometric head of lower aquifer, m

$H_{wl}$ = steady-state potentiometric head in borehole at lower aquifer, m

Using Darcy's equation, the steady-state, laminar, and isothermal flow flow of a homogeneous fluid through the borehole can be described by Equation 3:

$$Q = \pi r_w^2 \, K_w \frac{(H_{wu} - H_{wl})}{L} \tag{3}$$

where:

$K_w$ = hydraulic conductivity of borehole, m/yr

$L$ = length of borehole, m

Parameters Q, $H_{wu}$, and $H_{wl}$ are dependent, but Q will be the performance measure of interest. Figure 2-2 illustrates these parameters.

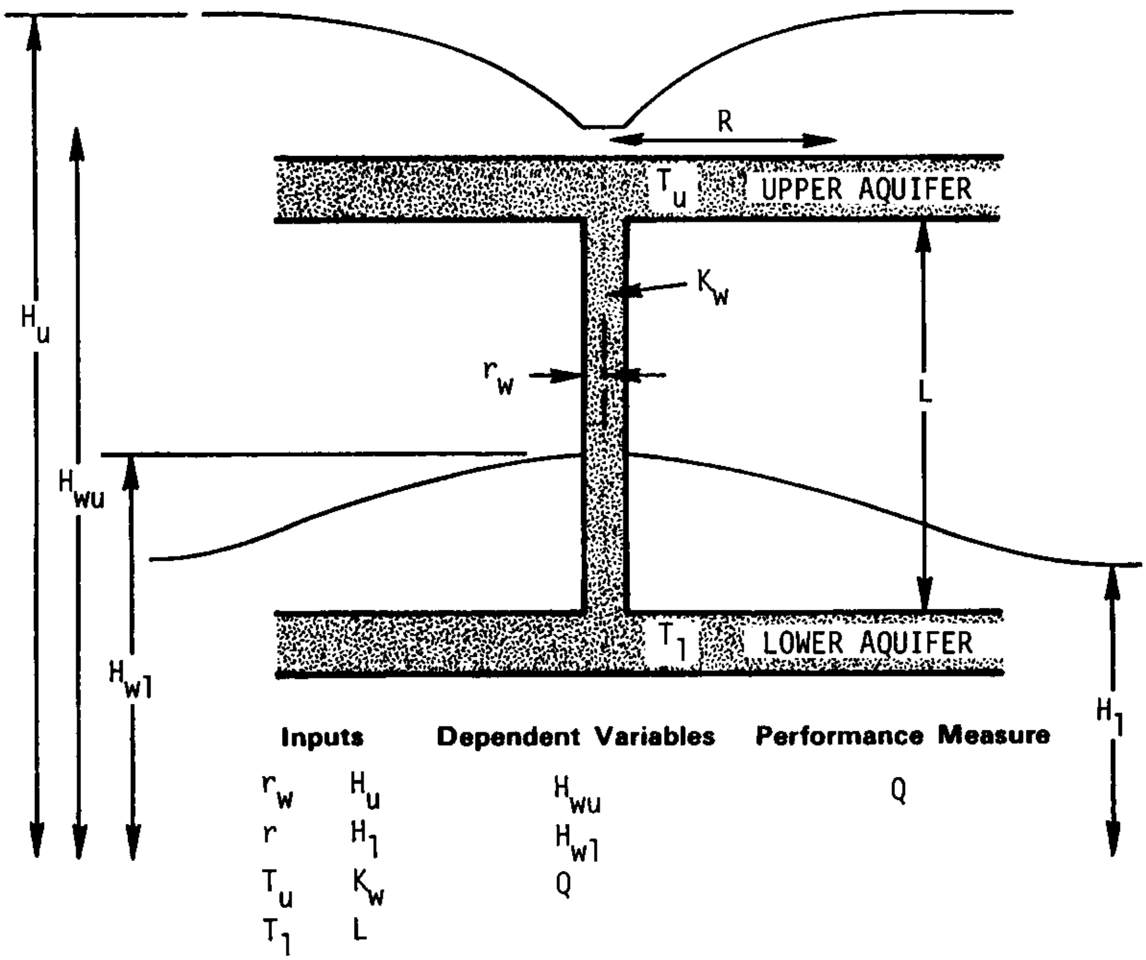


Figure 2-2. Borehole Scenario Parameters.

Attempts were made to define the probability distributions of the parameters using available information for typical salt sites. For those parameters where good statistical properties were not available, probability distributions were assumed and will be updated with the availability of additional data. At present, the distributions in Table 2-1 are assumed to be statistically independent implying zero pairwise covariance or correlation between the parameters. This assumption neglects the causal relationship of $K_w$ to $r_w$ and of r to transmissivity.

Table 2-1. Input Parameter Probability Distributions.

| Input Parameter | Range | Distribution |
|---|---|---|
| $r_w$ | 0.05 to 0.15 m | Normal ($\mu$ = 0.10, $\sigma$ = 0.0161812)* |
| $r$ | 100 to 50,000 m | Lognormal ($\mu'$ = 7.71, $\sigma'$ = 1.0056)** |
| $T_u$ | 63,070 to 111,600 $m^2$/yr | Uniform |
| $H_u$ | 990 to 1,110 m | Uniform |
| $T_l$ | 63.1 to 116 $m^2$/yr | Uniform |
| $H_l$ | 700 to 820 m | Uniform |
| $L$ | 1,120 to 1,680 m | Uniform |
| $K_w$ | 9,855 to 12,045 m/yr | Uniform |

*$\mu$, $\sigma$ are the mean and standard deviation, respectively, of $r_w$.

**$\mu'$, $\sigma'$ are the mean and standard deviation, respectively, of the ln (r) which is normally distributed.

Probability distributions of the input parameters in Table 2-1 were used to generate LHS input settings. They were also used to develop the covariance matrix needed for uncertainty analyses using deterministic sensitivity coefficients.

# 3 ANALYSIS TECHNIQUES

Two sensitivity/uncertainty analysis approaches are described to quantify the sensitivity of the performance measure (flow rate) to the input parameters and the contribution of each parameter to the overall uncertainty in flow rate. Latin Hypercube Sampling (LHS), as used by the U.S. Nuclear Regulatory Commission (NRC) in risk studies for nuclear waste repositories (Helton and Iman, 1980, and Pepping et al, 1983), and commonly associated statistical techniques are applied for the scenario analysis. The second approach is a deterministic evaluation of sensitivity coefficients plus statistical techniques to quantify the desired uncertainty.

As shown in this report, both approaches provide a suitable tool for sensitivity/uncertainty analysis of the hypothetical borehole scenario. It should be emphasized that, as in any analytic technique, no method is universally superior. The suitability of one method over others depends on the number of parameters, nonlinearity in the system, the size of the situation being analyzed, and the magnitude of uncertainty in the parameters.

## 3.1 STATISTICAL SAMPLING METHOD

The statistical sampling method described herein is based on the LHS technique. LHS is a member of a family of statistical sampling techniques which includes Monte Carlo sampling and stratified random sampling. LHS has been used in many risk assessments performed by Sandia National Laboratories for the NRC (Helton and Iman, 1980; Iman and Conover, 1980a,b; and Pepping et al, 1983).

### 3.1.1 Design Matrix Generation

Latin Hypercube Sampling is used to generate what is called a design matrix. Specifically, if N computer runs are to be made of the computer code to be analyzed with k parameters under study, the design matrix will be N x k. Each row of the matrix contains the input settings for each of the k parameters being studied. As explained in ONWI-444 (Harper, 1983), LHS has several potential advantages over the other sampling techniques (McKay et al, 1979) in that it covers the range of each input parameter and provides a method of introducing a desired rank correlation matrix for these parameters (Iman and Conover, 1980a).

Parameter probability density functions and any rank correlation between parameters must be specified so that this design matrix can be generated. Typically, for an N x k design matrix, each of the k parameters is divided into N equi-probable intervals. Then a random sample is chosen from each interval in a manner that preserves the individual probability density function. In this manner, k N-tuples (an N x 1 vector) of input settings are determined. If the parameters are all assumed to be statistically independent (implying the covariance matrix of the parameter is a diagonal matrix), then these N-tuples are randomly joined together to form the N x k design matrix. If independence of these parameters is not a reasonable assumption, then the desired rank correlation matrix specified is used to match the N-tuples in such a manner that approximates this correlation structure.

After the design matrix is established by LHS, other statistical techniques are used to establish the sensitivities and related uncertainty. It should be emphasized that LHS strictly ends with the development of the design matrix. In statistical parlance, it creates the experimental design. Since the design created is done so at random, it does not have nice properties such as orthogonality of classical experimental designs developed for hypothesis testing. (Classical screening designs usually sample only a few points in the range of values for each parameter (Hicks, 1973) as opposed to LHS covering the entire spectrum more fully.) Commonly, stepwise regression analysis and/or partial correlation are used in the identification of the key parameters, i.e., the most sensitive parameters.

The design matrix for N computer runs is used as input to the computer code of interest and the desired outputs (performance measures) are recorded for each of these N runs. The performance measures of interest can be studied as a function of the input parameters using techniques such as stepwise regression.

Often, the analysis is performed on the ranks of the data if highly nonlinear relationships are present between a performance measure and the inputs. The rank transformation replaces the raw data values by the integers 1, 2, ..., N if no ties exist between the values. If ties are present, the average rank is used for the tied values.

### 3.1.2 Borehole Scenario Analysis

Two sample sets of inputs were generated using LHS (Iman et al, 1980): one with 10 design points and the other with 50 design points. Each design point represents a separate combination of the eight parameters. The two design matrices created by LHS (N = 10 and N = 50) are shown in Appendix A.

The N = 50 design matrix was used as input to the borehole scenario flow computer code. For each of the 50 randomly generated design points, the computer code was run creating an output file that contained the design point and the corresponding value of performance measure Q. This output data set was then input into the MINITAB (Ryan et al, 1981) statistical software package. Various descriptive and inferential statistical techniques were used in the analysis of these data. The major technique utilized was stepwise regression. A summarized procedure for the stepwise regression technique is provided as Appendix B.

The parameter combination $H_u$ - $H_l$ was used as additional input to the stepwise regression analyses performed. Two models were evaluated. Model 1 is shown as Equation 4:

$$\hat{Q} = b_o + \sum_i b_i x_i + \sum_{ij}\sum b_{ij} x_i x_j \qquad \text{(Model 1)} \qquad (4)$$

where:

$\hat{Q}$ = predicted flow, $m^3/yr$

b = coefficient(s) of Model 1

x = parameters (eight initial plus $H_u$ - $H_l$)

Thus, quadratic and all cross-product terms are considered. Model 2 (linear terms only) is shown as Equation 5:

$$\hat{Q} = a_o + \sum_i a_i x_i \qquad \text{(Model 2)} \qquad (5)$$

where a = the coefficient(s) of Model 2. (Stepwise regression was performed considering both models for the raw data and the ranks of the data. Stepwise regression on the ranks did not provide a better fit than the raw data, so only the raw data were considered for the remainder of the analyses.)

Least squares estimates of coefficients (b or a) were obtained for the variables identified as significant in the stepwise regression procedure. The variability of the performance measure Q is divided into two parts. The first is the variability accounted for by the regression equation (Eq. 4 or 5), whereas, the remainder is the "residual" variability not covered by the regression equation. One measure of the "goodness of fit" of the regression is $R^2$, the percentage of variability of performance measure Q accounted for by the regression equation (or 100 times the ratio of the first part to the total variability of Q).

### 3.1.3 Analysis Results

Key parameters identified by this statistical sampling method, and the associated $R^2$ are shown (Table 3-1). The combination of variables given are parsimonious and are the "best" predictors for performance measure Q. Both models provide equally good fits to the data and involve the same variables.

Table 3-1. Key Parameters.

| Model 1 (allows quadratic and cross-product terms) | Model 2 (linear terms only) |
|---|---|
| $r_w(H_u-H_l)$ | $r_w$ |
| $(H_u-H_l)L$ | $H_u-H_l$ |
| $K_w$ | $L$ |
| | $K_w$ |
| $R^2 = 98.0$ | $R^2 = 97.6$ |

Table 3-2 presents estimated coefficients and the percent variability accounted for (from $R^2$) for the four parameters of Model 2. Model 2 will be used in the remainder of this report.

Table 3-2. LHS Model 2 Regression Estimates.

| Parameter | Estimated Coefficient | % Variance of Q |
|---|---|---|
| $r_w$ | 1405.0 | 70.5 |
| $H_u$-$H_l$ | 0.229 | 15.6 |
| L | -0.054 | 10.2 |
| $K_w$ | 0.005 | 1.3 |

## 3.2 DETERMINISTIC EVALUATION METHODS

Statistical methods for sensitivity analysis, such as those presented based on LHS, typically fit a polynomial to approximate the relationship between the desired output and the inputs. However, this relationship is explicitly detailed in the computer code itself. Deterministic approaches to sensitivity analysis take advantage of this fact. The two major deterministic approaches are the direct method and the adjoint method. The direct method is more appropriate when there are few parameters under study and many performance measures, whereas, the adjoint approach is superior when there are a limited number of performance measures and many parameters.

Obtaining parameter sensitivities using these deterministic methods is usually much more complicated than using statistical techniques. It requires differentiating the mathematical relationships used with respect to the performance measure of interest. ONWI-380 (Thomas, 1982), ONWI-444 (Harper, 1983), Oblow, 1978a,b, or Cacuci et al, 1982, describe in detail the process involved in deterministic evaluation methods. The mathematics used to obtain deterministic sensitivities for a simple borehole flow scenario is described. Partial derivatives and normalized sensitivity coefficients are obtained.

### 3.2.1 Mathematical Derivation

The steps given below show how deterministic sensitivity methods are applied to the borehole flow scenario analysis. Both the direct and adjoint methods are presented. The three governing equations (Eq. 1, 2, and 3) may be rewritten as follows:

$$Q = \frac{2\pi T_u \ (H_u - H_{wu})}{\ln \ (r/r_w)} = S_1 \ (H_u - H_{wu}) \tag{1'}$$

$$Q = - \frac{2\pi T_l \ (H_l - H_{wl})}{\ln \ (r/r_w)} = S_2 \ (H_l - H_{wl}) \tag{2'}$$

$$Q = \frac{\pi r_w^2 \ K_w \ (H_{wu} - H_{wl})}{L} = S_3 \ (H_{wu} - H_{wl}) \tag{3'}$$

The response or performance measure of interest is flow, Q, with $H_{wl}$ and $H_{wu}$ as additional dependent variables and the rest as independent parameters. It is now necessary to differentiate Q with respect to a generic parameter, $\alpha$, which can be any of the eight input parameters.

$$\frac{\partial Q}{\partial \alpha} = \frac{\partial S_1}{\partial \alpha} (H_u - H_{wu}) + S_1 \left( \frac{\partial H_u}{\partial \alpha} - \frac{\partial H_{wu}}{\partial \alpha} \right) \tag{6}$$

$$\frac{\partial Q}{\partial \alpha} = \frac{\partial S_2}{\partial \alpha} (H_l - H_{wl}) + S_2 \left( \frac{\partial H_l}{\partial \alpha} - \frac{\partial H_{wl}}{\partial \alpha} \right) \tag{7}$$

$$\frac{\partial Q}{\partial \alpha} = \frac{\partial S_3}{\partial \alpha} (H_{wu} - H_{wl}) + S_3 \left( \frac{\partial H_{wu}}{\partial \alpha} - \frac{\partial H_{wl}}{\partial \alpha} \right) \tag{8}$$

Letting $h_1 = \partial Q/\partial\alpha$, $h_2 = \partial H_{wl}/\partial\alpha$, and $h_3 = \partial H_{wu}/\partial\alpha$, and rearranging terms, we have Equations 9 through 11:

$$h_1 + S_1 h_3 = \frac{\partial S_1}{\partial\alpha}(H_u - H_{wu}) + S_1 \frac{\partial H_u}{\partial\alpha} \tag{9}$$

$$h_1 + S_2 h_2 = \frac{\partial S_2}{\partial\alpha}(H_1 - H_{wl}) + S_2 \frac{\partial H_{wl}}{\partial\alpha} \tag{10}$$

$$h_1 + S_3 h_2 - S_3 h_3 = \frac{\partial S_3}{\partial\alpha}(H_{wu} - H_{wl}) \tag{11}$$

or, Equation 12:

$$\underset{A}{\begin{pmatrix} 1 & 0 & S_1 \\ 1 & S_2 & 0 \\ 1 & S_3 & -S_3 \end{pmatrix}} \underset{h}{\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}} = \underset{S}{\begin{pmatrix} \frac{\partial S_1}{\partial\alpha}(H_u - H_{wu}) + S_1 \frac{\partial H_u}{\partial\alpha} \\ \frac{\partial S_2}{\partial\alpha}(H_1 - H_{wl}) + S_2 \frac{\partial H_1}{\partial\alpha} \\ \frac{\partial S_3}{\partial\alpha}(H_{wu} - H_{wl}) \end{pmatrix}} \tag{12}$$

At this point, we could solve for h as follows:

$$Ah = S \rightarrow h = A^{-1} S \tag{13}$$

(This is the direct method and results in partial derivatives for each of the three dependent variables, Q, $H_{wu}$, and $H_{wl}$, i.e., $h_1 = \partial Q/\partial\alpha$, $h_2 = \partial Q/\partial\alpha$, and $h_3 = \partial Q/\partial\alpha$.)

To solve for the desired sensitivities, partial derivatives for $S_1$, $S_2$, and $S_3$ must be specified for each $\alpha$ (the eight input parameters). These partial derivatives are provided in Table 3-3.

Table 3-3. Partial Derivatives Required for Deterministic Sensitivity Analysis.

| $\alpha$ | $\partial S_1/\partial\alpha$ | $\partial S_2/\partial\alpha$ | $\partial S_3/\partial\alpha$ |
|---|---|---|---|
| $T_u$ | $\frac{2\pi}{\ln(r/r_w)}$ | 0 | 0 |
| $H_u$ | 0 | 0 | 0 |
| $r$ | $-\frac{2\pi T_u}{r[\ln(r/r_w)]^2}$ | $\frac{2\pi T_1}{r[\ln(r/r_w)]^2}$ | 0 |
| $r_w$ | $\frac{2\pi T_u}{[\ln(r/r_w)]^2\ rr_w}$ | $\frac{-2\pi T_1}{[\ln(r/r_w)]^2\ rr_w}$ | $2\pi r_w K_w/L$ |
| $T_1$ | 0 | $\frac{-2\pi T_1}{r[\ln(r/r_w)]^2}$ | 0 |
| $H_1$ | 0 | 0 | 0 |
| $K_w$ | 0 | 0 | $\pi r_w^2/L$ |
| $L$ | 0 | 0 | $-\pi r_w^2 K_w/L^2$ |

The derivation of adjoint sensitivities is detailed. The response of interest will be denoted by R. Thus, R = Q, and we want to find the partial derivatives $\partial R/\partial \alpha$. Let $g' = (1, 0, 0)$ where the prime indicates a transpose. Then $g'h = h_1 = \partial Q/\partial \alpha = \partial R/\partial \alpha$. Given Eq. 12, multiply by an arbitrary row vector, $v'$, to obtain

$$v' \, (Ah-S) = 0 \tag{14}$$

Then $\partial R/\partial \alpha$ may be rewritten as

$$\frac{\partial R}{\partial \alpha} = g'h - v' \, (Ah-S) \tag{15}$$

or

$$\frac{\partial R}{\partial \alpha} = (g'-v'A)h + v'S \tag{16}$$

To remove h, set $(g'-v'A)$ to zero and solve for v:

$$g'-v'A = 0 \rightarrow v = (A')^{-1} \, g \tag{17}$$

Now the desired partials may be found from Equation 15 as

$$\frac{\partial R}{\partial \alpha} = v'S \tag{18}$$

Notice that we now may find $\partial R/\partial \alpha$ without needing to know the vector h. The vector S will change for each $\alpha$ using Table 4.

### 3.2.2 Results of Deterministic Based Analyses

The borehole flow program was modified to include the steps presented earlier for the development of direct and adjoint approaches. The results presented in Table 3-4 were based on a design point that was felt to be representative of the physical system. For the seven input parameters with symmetric distributions, the midpoint or mean of the range was selected. For r with a lognormal distribution, the geometric mean of the range was chosen. For this combination of values, the first partial derivatives ($\partial Q/\partial \alpha$) were derived as well as normalized sensitivity coefficients. Normalized sensitivity coefficients, $(\partial Q/\partial \alpha)(\alpha/Q)$, show the predicted percentage change in the performance measure, Q, for a given percentage change in $\alpha$.

Table 3-4. Deterministic Sensitivities.

| Parameter | Units | Value | $\partial Q/\partial \alpha$ | $(\partial Q/\partial \alpha)(\alpha/Q)$ |
|---|---|---|---|---|
| $T_u$ | $m^2/yr$ | 89,340.0 | $0.35 \times 10^{-5}$ | $0.44 \times 10^{-5}$ |
| $H_u$ | m | 1,050.0 | 0.245 | 3.62 |
| r | m | 2,236.0 | $-0.14 \times 10^{-4}$ | -0.0004 |
| $r_w$ | m | 0.1 | 1412.9 | 1.99 |
| $T_l$ | $m^2/yr$ | 89.55 | 0.003 | 0.004 |
| $H_l$ | m | 760.0 | -0.245 | -2.62 |
| $K_w$ | m/yr | 10,950.0 | 0.006 | 0.996 |
| L | m | 1,400.0 | -0.05 | -0.996 |

Since $H_u$ and $H_l$ by themselves are meaningless in this scenario, they can be combined into $H_u$-$H_l$. The partial, $\partial Q/\partial \alpha$, for $\alpha = H_u - H_l$ can be found from the following:

$$0.245\, H_u - 0.245\, H_l = 0.245\, (H_u - H_l) \rightarrow \frac{\partial Q}{\partial (H_u - H_l)} = 0.245 \tag{19}$$

The normalized sensitivity coefficient is found by adding the normalized sensitivity coefficient for $H_u$ and $H_l$. Thus,

$$\frac{\partial Q}{\partial (H_u - H_l)} \cdot \frac{H_u - H_l}{Q} = 3.62 - 2.62 = 1.00 \tag{20}$$

It can be shown, in general, that the normalized sensitivity coefficient for $H_u$-$H_l$ will always be 1 regardless of the individual values of $H_u$ and $H_l$. In a similar fashion, the normalized sensitivity coefficients of $K_w$ and A will always add to 0. In many situations, the normalized sensitivity coefficients are more useful than the first partials of the performance measure because one is often interested in the effect of percent changes in the parameters rather than absolute changes. For example, a selected percentage change in $r_w$ would cause roughly twice that percentage change in Q. Appendix D shows the stability of the normalized sensitivity coefficients over the ranges given in Table 2-1.

It is necessary to combine the partial derivatives with the parameter covariance matrix to estimate the uncertainty of the performance measure about the design point. This covariance matrix includes the parameter variance and correlation information and is combined with the partials as follows:

$$\hat{\sigma}^2_{pm} = \underset{\sim}{s}\Sigma\underset{\sim}{s}' \tag{21}$$

where:

$\hat{\sigma}^2_{pm}$ = uncertainty or variance estimate of the performance measure

$\underset{\sim}{s}$ = vector containing the first partial derivatives of performance measure

$\Sigma$ = parameter covariance matrix.

For our scenario, $\Sigma$ has a very simple structure due to the assumed independence of the parameters, i.e., $\Sigma$ is a diagonal matrix with parameter variances down the diagonal. Thus

$$\hat{\sigma}^2_{Q} = \underset{\sim}{s}\Sigma\underset{\sim}{s}' = \sum_i s_i \sigma_i^2 \tag{22}$$

where:

$s_i$ = element i of $\underset{\sim}{s}$

$\sigma_i^2$ = variance of parameter i

Carrying out the calculations of Eq. (21) and (22) and determining the percent of the variance accounted for by each parameter results are shown in Table 3-5 where $H_u$-$H_l$ is used in place of $H_u$ and $H_l$ separately.

Table 3-5. Estimated Performance Measure Uncertainty.

| Parameter | % Uncertainty |
|---|---|
| $r_w$ | 71.6 |
| $r$ | |
| $T_u$ | |
| $H_u$-$H_l$ | 17.9 |
| $T_l$ | |
| $L$ | 8.5 |
| $K_w$ | 2.0 |

# 4 COMPARATIVE ANALYSIS RESULTS

## 4.1 INITIAL METHODS COMPARISON

At this point, it is worthwhile to compare the results obtained by the LHS approach and the deterministic approach. This initial comparison is based on the N = 50 sample for LHS and the 1 design point for the deterministic. Tables 4-1 and 4-2 summarize the results of this comparison.

Table 4-1. LHS Approach Analysis Results.

| Parameter | Units | Stepwise Regression Ranking | Regression Coefficient | % Uncertainty |
|---|---|---|---|---|
| $r_w$ | m | 1 | 1,405.0 | 70.5 |
| $r$ | m | | | |
| $T_u$ | $m^2/yr$ | | | |
| $H_u-H_l$ | m | 2 | 0.229 | 15.6 |
| $T_l$ | $m^2/yr$ | | | |
| L | m | 3 | -0.054 | 10.2 |
| $K_w$ | m/yr | 4 | 0.005 | 1.3 |

A comparison of Table 4-1 and 4-2 finds very similar results for both approaches. Deterministic sensitivity usually uses the absolute value of the normalized sensitivity coefficients to rank the parameters in order of importance. Doing this results in $r_w$ being ranked first, $H_u-H_l$ second, with L and $K_w$ tied for third. This closely parallels the results of the LHS approach. The regression coefficient estimates obtained by the LHS approach are very close to the first partial derivatives obtained by the direct and adjoint techniques. Also, the percent uncertainty for the parameters are in close agreement for the two methods.

Table 4-2. Deterministic Approach Analysis Results.

| Parameter | Units | Normalized Sensitivity Coefficient | First Partial Derivative | % Uncertainty |
|---|---|---|---|---|
| $r_w$ | m | 1.99 | 1412.9 | 71.6 |
| r | m | -0.00044 | $-0.14 \times 10^{-4}$ | |
| $T_u$ | $m^2/yr$ | 0.000004 | $0.35 \times 10^{-8}$ | |
| $H_u$-$H_1$ | m | 1.0 | 0.245 | 17.9 |
| $T_1$ | $m^2/yr$ | 0.0044 | 0.003 | |
| L | m | -0.996 | -0.05 | 8.5 |
| $K_w$ | m/yr | 0.996 | 0.006 | 2.0 |

Summarizing this, we see very similar results for both techniques on this relatively simple computer code to model flow through a borehole. The LHS approach is much simpler to apply for one familiar with standard statistical techniques; however, the direct and adjoint results presented used only one run of the computer code to find all the sensitivities. If quick turnaround is needed on a new code and there are a moderate number of parameters, LHS can be used to identify the key parameters. For a large problem with thousands of parameters, the adjoint approach provides a comprehensive screening of all parameters that is not possible with statistical techniques alone. However, analytic solution of the adjoint equations may take many person-months to complete.

As explained on ONWI-444 (Harper, 1983), ONWI needs capabilities such as the adjoint technique to provide a comprehensive screening of all parameters for licensing. To expedite the time it takes to develop an adjoint version of the code, Oak Ridge National Laboratory will develop a compiler called GRESS (Oblow, 1983a). The Gradient-Enhanced Software System (GRESS) uses computer calculus (Oblow, 1983b) on a FORTRAN-based computer code to greatly reduce the time and cost of developing adjoint codes. As GRESS evolves, deterministic sensitivity procedures can be used in situations where one cannot afford to wait a year or longer for the necessary mathematics to be performed analytically on complex codes.

The major part of the initial comparisons used the 50 design point set for the LHS approach; whereas 1 design point was used for the deterministic methods. However, the close agreement found betweem the methods moved us to consider further comparisons.

## 4.2 ADDITIONAL COMPARISONS

The LHS and deterministic methods on both the N = 10 and N = 50 design matrices generated by LHS are compared (Table 4-3). This is done strictly to compare the sensitivity coefficients obtained by the two methods. In practice, one would not run the adjoint code for 50 design points; however, multiple design points may be needed for highly nonlinear codes in which the parameters exhibit high variability. These two design matrices and the corresponding dependent variable values may be found in Appendix A. Appendix C gives the deterministic sensitivity results for the N = 10 case.

Closer agreement is found between the N = 50 comparison of deterministic and LHS sensitivity coefficients than the N = 10 comparison though both are within reasonable agreement. For the N = 10 case, $K_w$ was not found to be statistically significant by stepwise regression. Since a model with linear terms only for the independent parameters provided a good fit to the data using regression techniques, this comparison between the two approaches makes sense. When quadratic and cross-product terms enter the model, such a simple comparison is not feasible. In that case, when one changes the value of a parameter, one must consider the impact of any quadratic and cross-product terms when predicting the new value of the performance measure.

Table 4-3. Further Comparisons of Methods.

| Parameter | Units | N = 50 | | N = 10 | |
|---|---|---|---|---|---|
| | | LHS Regression Coefficient | Average Deterministic Partial Derivative | LHS Regression Coefficient | Average Deterministic Partial Derivative |
| $r_w$ | m | 1405.0 | 1434.0 | 1484.0 | 1440.0 |
| $r$ | m | | $-0.287 \times 10^{-4}$ | | $-0.259 \times 10^{-4}$ |
| $T_u$ | $m^2/yr$ | | $0.458 \times 10^{-8}$ | | $0.435 \times 10^{-8}$ |
| $H_u$-$H_l$ | m | 0.229 | 0.255 | 0.216 | 0.252 |
| $T_l$ | $m^2/yr$ | | 0.005 | | 0.004 |
| L | m | -0.054 | -0.054 | -0.06 | -0.054 |
| $K_w$ | m/yr | 0.005 | 0.007 | | 0.007 |

## 4.3 CUMULATIVE DISTRIBUTION FUNCTION

When using the LHS approach, it is possible to develop an empirical probability distribution for the performance measure of interest. If the assumed input parameter distributions and correlations are correct, and the computer code is a valid representation of the relationship between the inputs and outputs, then the empirical probability distribution should provide a good estimate of the true probability distribution of the performance measure. A cumulative distribution function (CDF) often provides more useful risk information. Estimates of the probability of being below a specific flow rate for Q may be read directly off the empirical CDF seen in Figure 4-1. Some individuals prefer using a complementary cumulative distribution function (CCDF) as seen in Figure 4-2. This allows one to directly obtain probability estimates of exceeding a certain flow rate rather than having to compute it from the CDF.

To obtain representative CDF and CCDF, it is necessary to use a Monte Carlo sampling method such as LHS to determine the input settings. While deterministic methods may be coupled with sampling methods (as described in Section 4.2), usually not enough computer runs are performed to develop good empirical CDF. This is a plus from an efficiency point of view for sensitivity analysis; however, it is generally insufficient for the establishment of an empirical CDF. If an emperical CDF was an important part of the analysis being performed, one must make sure to couple it with a sampling approach to get the design points and to perform a sufficient number of computer runs to result in a reasonable estimate of the true CDF.

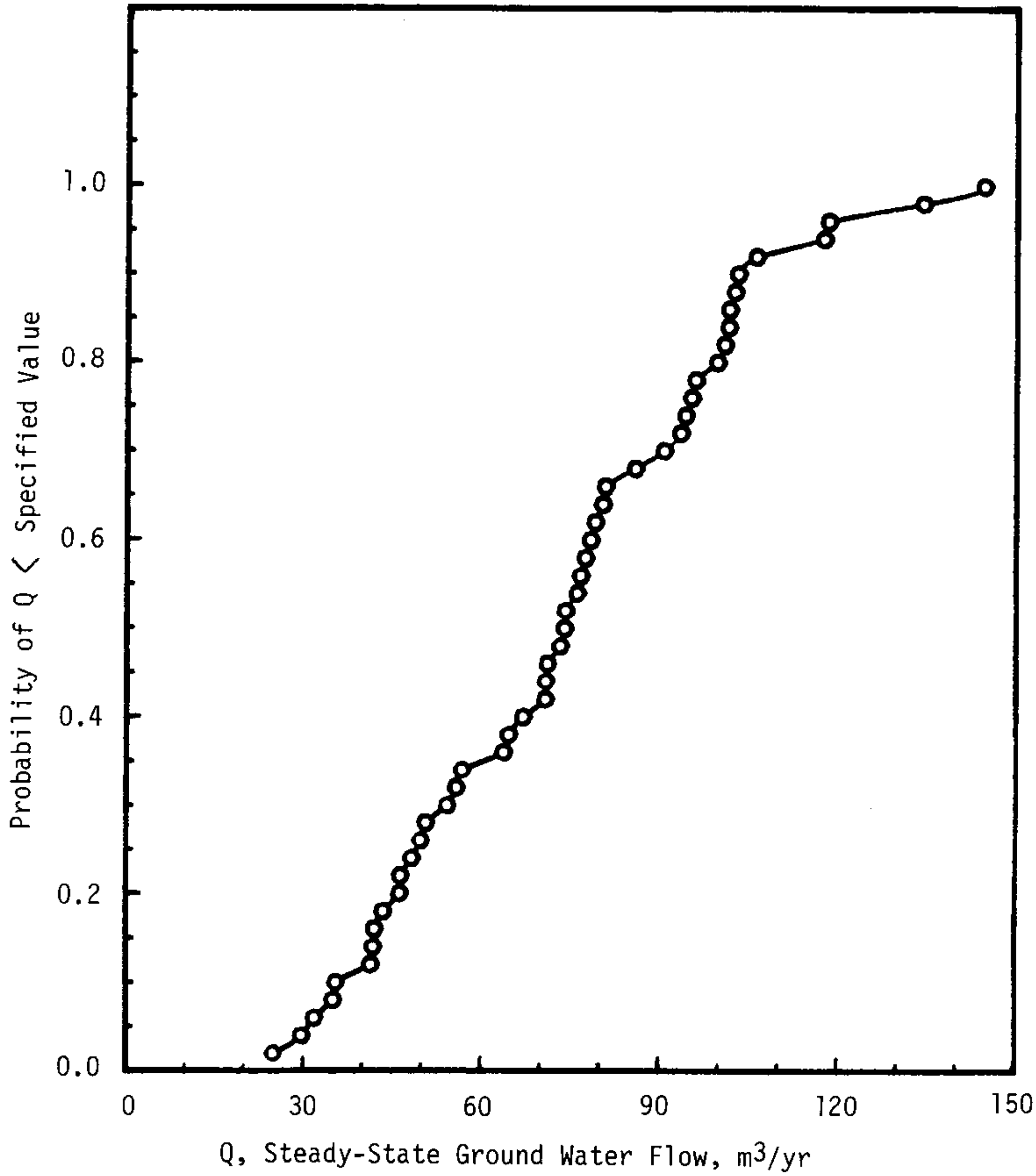


Figure 4-1. Empirical Cumulative Distribution Function for Q.

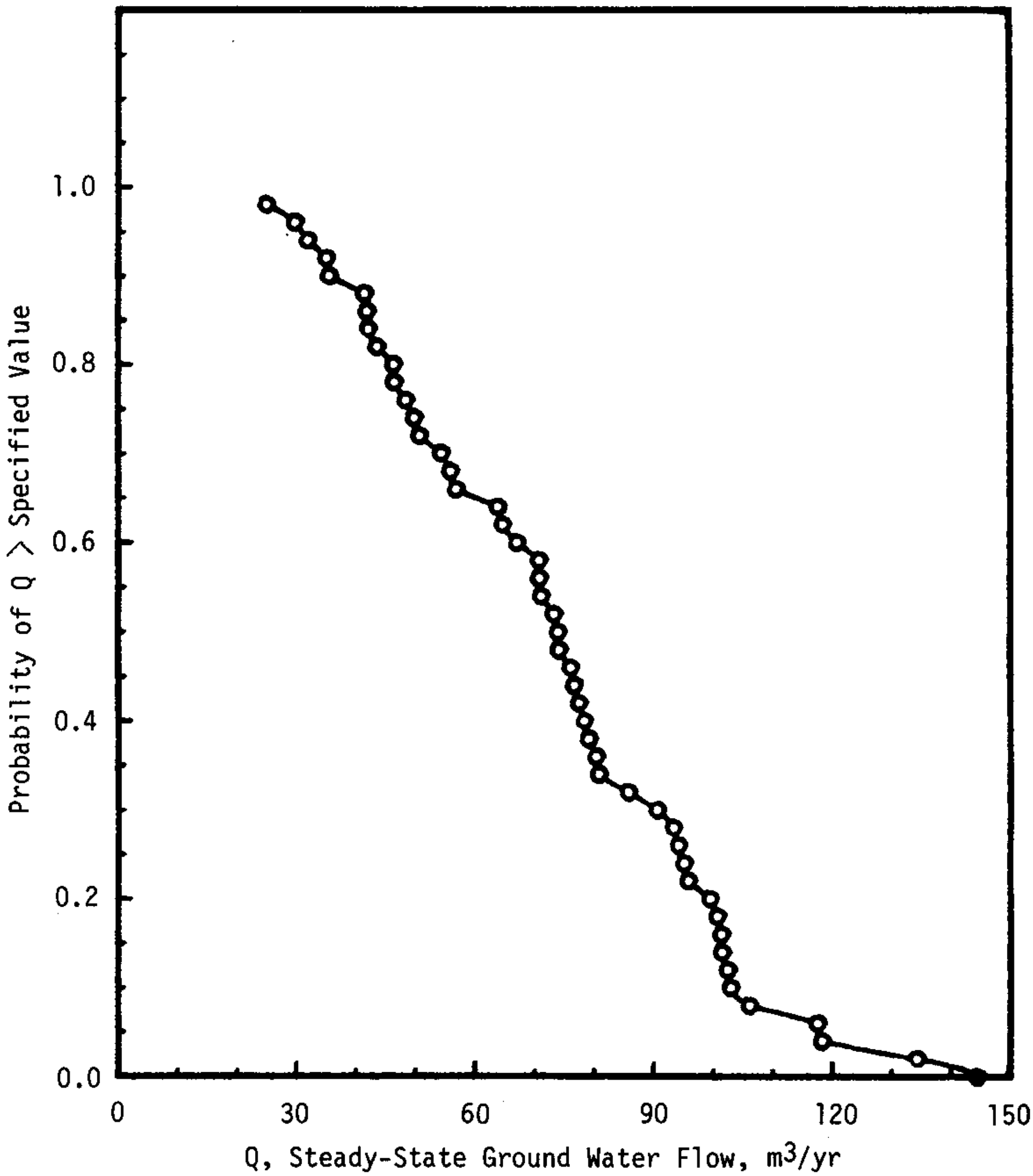


Figure 4-2. Empirical Complementary Cumulative Distribution Function for Q.

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# APPENDIX A - LATIN HYPERCUBE SAMPLING DESIGN MATRICES

In this appendix, the Latin Hypercube Sampling (LHS) design matrices are presented (Table A-1 and A-2) for the two cases discussed, N = 10 and N = 50. In addition, the three dependent variables ( Q, $H_{wu}$, and $H_{wl}$) have their resultant values given for each of the design points. The variables are presented in the column order indicated below:

| | | |
|---|---|---|
| $r_w$ | m | Values generated by LHS |
| $r$ | m | |
| $T_u$ | $m^2/yr$ | |
| $H_u$ | m | |
| $T_l$ | $m^2/yr$ | |
| $H_l$ | m | |
| $L$ | m | |
| $K_w$ | m/yr | |
| $Q$ | $m^3/yr$ | Values generated by running borehole flow computer code for each of N design points |
| $H_{wu}$ | m | |
| $H_{wl}$ | m | |

Table A-1. LHS Boreflow Results, N = 10.

| Run No. | $r_w$ | $r$ | $T_u$ | $H_u$ | $T_l$ | $H_l$ | $L$ | $K_w$ | $H_{wl}$ | $H_{wu}$ | $Q$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.8609E-01 | 2948.0 | 0.8337E+05 | 1044.0 | 107.0 | 783.0 | 1250.0 | 0.1001E+05 | 783.754 | 1044.0 | 48.5247 |
| 2 | 0.1050 | 5194.0 | 0.8840E+05 | 1093.0 | 67.7 | 788.0 | 1181.0 | 0.1126E+05 | 790.538 | 1093.0 | 99.8815 |
| 3 | 0.1180 | 1358.0 | 0.9471E+05 | 993.0 | 98.6 | 758.0 | 1466.0 | 0.1174E+05 | 759.236 | 992.999 | 81.8886 |
| 4 | 0.9050E-01 | 240.0 | 0.1091E+06 | 1037.0 | 81.2 | 811.0 | 1534.0 | 0.1052E+05 | 811.614 | 1037.0 | 39.7706 |
| 5 | 0.9287E-01 | 1861.0 | 0.9219E+05 | 1101.1 | 103.0 | 711.0 | 1575.0 | 0.1075E+05 | 712.101 | 1101.0 | 71.9224 |
| 6 | 0.1250 | 1165.0 | 0.1015E+06 | 1055.0 | 86.5 | 734.0 | 1325.0 | 0.1023E+05 | 736.033 | 1055.0 | 120.885 |
| 7 | 0.9778E-01 | 902.0 | 0.6798E+05 | 1072.0 | 76.9 | 715.0 | 1380.0 | 0.1116E+05 | 716.631 | 1072.0 | 86.3202 |
| 8 | 0.1100 | 2616.0 | 0.6944E+05 | 1085.0 | 113.0 | 799.0 | 1429.0 | 0.1147E+05 | 800.233 | 1085.0 | 86.8866 |
| 9 | 0.1000 | 14690.0 | 0.7427E+05 | 1009.0 | 70.8 | 768.0 | 1672.0 | 0.1043E+05 | 769.257 | 1009.0 | 46.9832 |
| 10 | 0.7457E-01 | 8017.0 | 0.1153E+06 | 1025.0 | 90.6 | 738.0 | 1141.0 | 0.1202E+05 | 739.071 | 1025.0 | 52.6204 |

Table A-2. LHS Boreflow Results, N = 50.

| Run No. | $r_w$ | $r$ | $T_u$ | $H_u$ | $T_l$ | $H_l$ | $L$ | $K_w$ | $H_{wl}$ | $H_{wu}$ | $Q$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.1200 | 307.0 | 0.7133E+05 | 1070.0 | 87.4 | 704.0 | 1504.0 | 0.1080E+05 | 705.691 | 1070.0 | 118.347 |
| 2 | 0.1210 | 7787.0 | 0.8976E+05 | 1088.0 | 88.5 | 812.0 | 1672.0 | 0.1071E+05 | 813.610 | 1089.0 | 80.8426 |
| 3 | 0.1040 | 2684.0 | 0.7764E+05 | 1092.0 | 91.4 | 782.0 | 1245.0 | 0.1120E+05 | 783.667 | 1092.0 | 94.2502 |
| 4 | 0.7928E-01 | 455.0 | 0.6463E+05 | 1046.0 | 63.1 | 801.0 | 1556.0 | 0.1147E+05 | 801.776 | 1046.0 | 35.5482 |
| 5 | 0.1170 | 480.0 | 0.9364E+05 | 1034.0 | 64.2 | 730.0 | 1430.0 | 0.1129E+05 | 732.114 | 1034.0 | 102.499 |
| 6 | 0.1010 | 1508.0 | 0.1029E+06 | 1006.0 | 114.0 | 737.0 | 1590.0 | 0.1051E+05 | 737.762 | 1006.0 | 56.8220 |
| 7 | 0.1130 | 16040.0 | 0.1038E+06 | 1004.0 | 103.0 | 714.0 | 1352.0 | 0.1179E+05 | 715.848 | 1004.0 | 100.801 |
| 8 | 0.1120 | 1121.0 | 0.8194E+05 | 1036.0 | 105.0 | 702.0 | 1282.0 | 0.1008E+05 | 703.439 | 1036.0 | 103.045 |
| 9 | 0.1000 | 24370.0 | 0.1060E+06 | 1023.0 | 84.1 | 711.0 | 1480.0 | 0.1177E+05 | 712.819 | 1023.0 | 77.4957 |
| 10 | 0.8917E-01 | 3967.0 | 0.1044E+06 | 1063.0 | 82.1 | 750.0 | 1154.0 | 0.9943E+04 | 751.392 | 1063.0 | 67.0667 |
| 11 | 0.9504E-01 | 1798.0 | 0.7483E+05 | 997.0 | 95.3 | 753.0 | 1138.0 | 0.1067E+05 | 754.063 | 996.999 | 64.6362 |
| 12 | 0.7425E-01 | 4686.0 | 0.1081E+06 | 1017.0 | 104.0 | 778.0 | 1311.0 | 0.1014E+05 | 778.540 | 1017.0 | 31.9442 |
| 13 | 0.9985E-01 | 1985.0 | 0.7907E+05 | 1044.0 | 71.2 | 725.0 | 1211.0 | 0.1106E+05 | 727.006 | 1044.0 | 90.6787 |
| 14 | 0.9564E-01 | 750.0 | 0.9121E+05 | 1025.0 | 113.0 | 793.0 | 1544.0 | 0.1156E+05 | 793.629 | 1025.0 | 49.7792 |
| 15 | 0.1150 | 6951.0 | 0.9751E+05 | 1008.0 | 69.2 | 799.0 | 1523.0 | 0.9870E+04 | 800.415 | 1008.0 | 55.8928 |
| 16 | 0.9209E-01 | 11710.0 | 0.1019E+06 | 1109.0 | 80.9 | 723.0 | 1525.0 | 0.1198E+05 | 724.859 | 1109.0 | 80.3990 |
| 17 | 0.1100 | 274.0 | 0.1120E+06 | 1039.0 | 112.0 | 745.0 | 1257.0 | 1.1075E+05 | 746.058 | 1039.0 | 95.2331 |
| 18 | 0.7821E-01 | 3134.0 | 0.8075E+05 | 1009.0 | 73.7 | 734.0 | 1651.0 | 0.1100E+05 | 734.803 | 1009.0 | 35.1060 |
| 19 | 0.7562E-01 | 2346.0 | 0.1071E+06 | 993.0 | 81.3 | 810.0 | 1303.0 | 0.1185E+05 | 810.603 | 993.0 | 29.7998 |
| 20 | 0.1080 | 2134.0 | 0.1134E+06 | 1072.0 | 103.0 | 819.0 | 1392.0 | 0.1116E+05 | 820.131 | 1072.0 | 73.9938 |
| 21 | 0.1030 | 2869.0 | 0.7306E+05 | 1014.0 | 110.0 | 813.0 | 1623.0 | 0.1017E+05 | 813.620 | 1014.0 | 41.8485 |
| 22 | 0.1040 | 10170.0 | 0.8365E+05 | 1079.0 | 100.0 | 727.0 | 1453.0 | 0.1171E+05 | 728.754 | 1079.0 | 95.9132 |
| 23 | 0.8143E-01 | 1359.0 | 0.6610E+05 | 1053.0 | 86.9 | 747.0 | 1500.0 | 0.1141E+05 | 747.861 | 1053.0 | 48.3514 |
| 24 | 0.1300 | 6148.0 | 0.9541E+05 | 1058.0 | 95.7 | 781.0 | 1179.0 | 0.1086E+05 | 783.404 | 1058.0 | 134.290 |
| 25 | 0.9691E-01 | 840.0 | 0.9264E+05 | 992.0 | 75.2 | 816.0 | 1226.0 | 0.1204E+05 | 816.973 | 991.999 | 50.7137 |

Table A-2. LHS Boreflow Results, N = 50 (Continued).

| Run No. | $r_w$ | r | $T_u$ | $H_u$ | $T_l$ | $H_l$ | L | $K_w$ | $H_{wl}$ | $H_{wu}$ | Q |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 26 | 0.9251E-01 | 664.0 | 0.1009E+06 | 1099.0 | 90.2 | 785.0 | 1379.0 | 0.1166E+05 | 786.114 | 1099.0 | 71.1289 |
| 27 | 0.9083E-01 | 2403.0 | 0.8688E+05 | 1104.0 | 107.0 | 795.0 | 1602.0 | 0.1089E+05 | 795.822 | 1104.0 | 54.2967 |
| 28 | 0.8480E-01 | 2023.0 | 0.9936E+05 | 1088.0 | 106.0 | 720.0 | 1235.0 | 0.1134E+05 | 721.152 | 1088.0 | 76.0979 |
| 29 | 0.1110 | 985.0 | 0.9662E+05 | 1052.0 | 77.4 | 754.0 | 1636.0 | 0.1093E+05 | 755.434 | 1052.0 | 76.6925 |
| 30 | 0.1180 | 1720.0 | 0.1149E+06 | 1012.0 | 76.3 | 716.0 | 1406.0 | 0.1161E+05 | 718.123 | 1012.0 | 106.151 |
| 31 | 0.8566E-01 | 4939.0 | 0.9829E+05 | 995.0 | 69.8 | 742.0 | 1298.0 | 0.1036E+05 | 743.158 | 994.999 | 46.3359 |
| 32 | 0.1050 | 1293.0 | 0.8778E+05 | 1101.0 | 108.0 | 797.0 | 1478.0 | 0.1032E+05 | 798.017 | 1101.0 | 73.2740 |
| 33 | 0.8969E-01 | 3494.0 | 0.1139E+06 | 1042.0 | 101.0 | 790.0 | 1601.0 | 0.1061E+05 | 790.701 | 1042.0 | 42.0874 |
| 34 | 0.1090 | 1894.0 | 0.6707E+05 | 1061.0 | 84.4 | 789.0 | 1437.0 | 0.1126E+05 | 790.457 | 1061.0 | 79.1258 |
| 35 | 0.1250 | 4345.0 | 0.6927E+05 | 1021.0 | 78.6 | 762.0 | 1669.0 | 0.1135E+05 | 763.818 | 1021.0 | 85.8513 |
| 36 | 0.9419E-01 | 4163.0 | 0.9238E+05 | 1107.0 | 66.7 | 804.0 | 1123.0 | 0.1049E+05 | 806.000 | 1107.0 | 78.3645 |
| 37 | 0.9908E-01 | 779.0 | 0.1102E+06 | 1084.0 | 65.3 | 758.0 | 1569.0 | 0.1000E+05 | 759.395 | 1084.0 | 63.8048 |
| 38 | 0.1150 | 5981.0 | 0.7600E+05 | 1077.0 | 92.9 | 807.0 | 1151.0 | 0.1029E+05 | 808.853 | 1077.0 | 99.5990 |
| 39 | 0.1270 | 1220.0 | 0.7380E+05 | 1049.0 | 91.6 | 769.0 | 1207.0 | 0.1007E+05 | 770.873 | 1049.0 | 117.576 |
| 40 | 0.6327E-01 | 3603.0 | 0.7221E+05 | 1032.0 | 73.0 | 740.0 | 1459.0 | 0.9940E+04 | 740.596 | 1032.0 | 24.9673 |
| 41 | 0.8654E-01 | 1035.0 | 0.8923E+05 | 1000.0 | 99.0 | 774.0 | 1336.0 | 0.1097E+05 | 774.657 | 999.999 | 43.5338 |
| 42 | 0.8840E-01 | 1412.0 | 0.8486E+05 | 1083.0 | 111.0 | 775.0 | 1171.0 | 0.1152E+05 | 776.029 | 1083.0 | 74.1388 |
| 43 | 0.1380 | 2896.0 | 0.8553E+05 | 1028.0 | 79.1 | 731.0 | 1267.0 | 0.1041E+05 | 733.895 | 1028.0 | 144.571 |
| 44 | 0.9766E-01 | 3368.0 | 0.7792E+05 | 1075.0 | 97.0 | 709.0 | 1632.0 | 0.1057E+05 | 710.214 | 1075.0 | 70.7905 |
| 45 | 0.1070 | 910.0 | 0.8256E+05 | 1067.0 | 71.9 | 765.0 | 1377.0 | 0.1192E+05 | 766.872 | 1067.0 | 93.4468 |
| 46 | 0.9668E-01 | 5456.0 | 0.6340E+05 | 1066.0 | 98.4 | 718.0 | 1191.0 | 0.1190E+05 | 719.797 | 1066.0 | 101.575 |
| 47 | 0.1070 | 1569.0 | 0.1083E+06 | 1096.0 | 67.7 | 771.0 | 1328.0 | 0.1161E+05 | 773.289 | 1096.0 | 101.476 |
| 48 | 0.8296E-01 | 8875.0 | 0.6732E+05 | 1056.0 | 85.6 | 757.0 | 1420.0 | 0.1024E+05 | 758.000 | 1056.0 | 46.4636 |
| 49 | 0.1020 | 2523.0 | 0.7028E+05 | 1031.0 | 115.0 | 763.0 | 1365.0 | 0.1108E+05 | 763.992 | 1031.0 | 70.8401 |
| 50 | 0.7149E-01 | 605.0 | 0.1109E+06 | 1093.0 | 93.6 | 706.0 | 1562.0 | 0.1044E+05 | 706.638 | 1093.0 | 41.4624 |

## APPENDIX B - STEPWISE REGRESSION

Stepwise regression is one of several statistical techniques commonly employed to identify the "best" subset of parameters that can be used to predict a performance measure of interest (e.g., Q). The procedure for this technique may be summarized as follows:

1. Select the variable (parameter by itself, crossproduct, or higher order term depending on assumed model for Q) with the largest absolute linear correlation with Q.

2. Using partial correlation (instead of the simple linear correlation in step 1), select the variable with the highest absolute partial correlation. This partial correlation with Q accounts for the variables already entered into the stepwise model being formed. If this variable's partial correlation meets a specified entry significance level, enter it into the model; otherwise, terminate the stepwise regression.

3. Given the variable in step 2 that just entered the regression equation, calculate the partial correlation of all variables previously entered into the model. If any of them no longer meet a specified removal significance level, delete them from the regression equation.

4. Go to step 2.

# APPENDIX C - DETERMINISTIC SENSITIVITY COEFFICIENTS FOR THE N = 10 DESIGN MATRIX

This appendix presents the detailed results of the deterministic sensitivity analysis based on the N = 10 LHS generaged design matrix (Table C-1). For each of the 10 design points, the three dependent variables (Q, $H_{wl}$, and $H_{wu}$) are listed at the top. The eight input parameters follow with their values (generated by LHS), first partial derivative sensitivity coefficient ($\partial R/\partial\alpha$), and normalized sensitivity coefficient $[(\partial R/\partial\alpha)(\alpha/R)]$.

Table C-1. Detailed Results of the Deterministic Sensitivity Analysis.

| Parameter | Value | $\frac{\partial R}{\partial \alpha}$ | $\frac{\partial R}{\partial \alpha}\frac{\alpha}{R}$ |
|---|---|---|---|
| 1 | R=Q = 48.52464 | $H_{wl}$ = 783.7536 | $H_{wu}$ = 1043.999 |
| $T_u$ | 83370.00 | 0.2156935E-08 | 0.3705821E-05 |
| $H_u$ | 1044.00 | 0.1859182 | 4.000000 |
| r | 2948.00 | -0.4557747E-05 | -0.2758951E-03 |
| $r_w$ | 0.08609 | 1124.082 | 1.994291 |
| $T_l$ | 107.00 | 0.1309451E-02 | 0.2887424E-02 |
| $H_l$ | 783.00 | -0.1859182 | -3.000000 |
| $K_w$ | 10010.00 | 0.4833502E-02 | 0.9971089 |
| L | 1250.00 | -0.3870748E-01 | -0.9971089 |
| 2 | R=Q = 99.88144 | $H_{wl}$ = 790.5381 | $H_{wu}$ = 1092.998 |
| $T_u$ | 88400.00 | 0.7200674E-08 | 0.6372952E-05 |
| $H_u$ | 1093.00 | 0.3274801 | 3.583607 |
| r | 5194.00 | -0.1481603E-04 | -0.7704580E-03 |
| $r_w$ | 0.105 | 1886.844 | 1.983538 |
| $T_l$ | 67.70 | 0.1227723E-01 | 0.8321550E-02 |
| $H_l$ | 788.00 | -0.3274801 | -2.583607 |
| $K_w$ | 11260.00 | 0.8796593E-02 | 0.9916721 |
| L | 1181.00 | -0.8386930E-01 | -0.9916721 |

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Table C-1. (Continued).

| Parameter | Value | $\frac{\partial R}{\partial \alpha}$ | $\frac{\partial R}{\partial \alpha}\frac{\alpha}{R}$ |
|---|---|---|---|
| 3 | R=Q = 81.88855 | $H_{wl}$ = 759.2360 | $H_{wu}$ = 992.9987 |
| $T_u$ | 94710.00 | 0.4734320E-08 | 0.5475582E-05 |
| $H_u$ | 993.00 | 0.3484619 | 4.225532 |
| $r$ | 1358.00 | -0.3395267E-04 | -0.5630546E-03 |
| $r_w$ | 0.118 | 1380.910 | 1.989868 |
| $T_l$ | 98.60 | 0.4368129E-02 | 0.5259557E-02 |
| $H_l$ | 758.00 | -0.3484619 | -3.225532 |
| $K_w$ | 11740.00 | 0.6938450E-02 | 0.9947350 |
| $L$ | 1466.00 | -0.5556440E-01 | -0.9947350 |
| 4 | R=Q = 39.77062 | $H_{wl}$ = 811.6145 | $H_{wu}$ = 1037.000 |
| $T_u$ | 109100.00 | 0.7377022E-09 | 0.2023687E-05 |
| $H_u$ | 1037.00 | 0.1759762 | 4.588496 |
| $r$ | 240.00 | -0.5719953E-04 | -0.3451766E-03 |
| $r_w$ | 0.0905 | 877.3659 | 1.996489 |
| $T_l$ | 81.20 | 0.1331737E-02 | 0.2719018E-02 |
| $H_l$ | 811.00 | -0.1759762 | -3.588496 |
| $K_w$ | 10520.00 | 0.3770191E-02 | 0.9972790 |
| $L$ | 1534.00 | -0.2585555E-01 | -0.9972790 |
| 5 | R=Q = 71.92241 | $H_{wl}$ = 712.1008 | $H_{wu}$ = 1100.999 |
| $T_u$ | 92190.00 | 0.2460308E-08 | 0.3153618E-05 |
| $H_u$ | 1101.00 | 0.1844164 | 2.823077 |
| $r$ | 1861.00 | -0.1102517E-04 | -0.2852775E-03 |
| $r_w$ | 0.09287 | 1544.613 | 1.994485 |
| $T_l$ | 103.00 | 0.1970982E-02 | 0.2822641E-02 |
| $H_l$ | 711.00 | -0.1844164 | -1.823077 |
| $K_w$ | 10750.00 | 0.6671550E-02 | 0.9971741 |
| $L$ | 1575.00 | -0.4553598E-01 | -0.9971741 |
| 6 | R=Q = 120.8852 | $H_{wl}$ = 736.0329 | $H_{wu}$ = 1054.998 |
| $T_u$ | 101500.00 | 0.6427935E-08 | 0.5397149E-05 |
| $H_u$ | 1055.00 | 0.3765893 | 3.286604 |
| $r$ | 1165.00 | -0.7195967E-04 | -0.6934929E-03 |
| $r_w$ | 0.125 | 1922.578 | 1.988021 |
| $T_l$ | 86.50 | 0.8850572E-02 | 0.6333071E-02 |
| $H_l$ | 734.00 | -0.3765893 | -2.286604 |
| $K_w$ | 10230.00 | 0.1174183E-01 | 0.9936614 |
| $L$ | 1325.00 | -0.9065580E-01 | -0.9936614 |

Table C-1. (Continued).

| Parameter | Value | $\frac{\partial R}{\partial \alpha}$ | $\frac{\partial R}{\partial \alpha}\frac{\alpha}{R}$ |
|---|---|---|---|
| 7 | R=Q = 86.32016 | $H_{wl}$ = 716.6310 | $H_{wu}$ = 1071.998 |
| $T_u$ | 67980.00 | 0.6562483E-08 | 0.5168174E-05 |
| $H_u$ | 1072.00 | 0.2417931 | 3.002801 |
| r | 902.00 | -0.4794403E-04 | -0.5009897E-03 |
| $r_w$ | 0.09778 | 1757.957 | 1.991343 |
| $T_l$ | 76.90 | 0.5128352E-02 | 0.4568693E-02 |
| $H_l$ | 715.00 | -0.2417931 | -2.002801 |
| $K_w$ | 11160.00 | 0.7699403E-02 | 0.9954261 |
| L | 1380.00 | -0.6226474E-01 | -0.9954261 |
| 8 | R=Q = 86.88659 | $H_{wl}$ = 800.2331 | $H_{wu}$ = 1084.998 |
| $T_u$ | 69440.00 | 0.8779252E-08 | 0.7016403E-05 |
| $H_u$ | 1085.00 | 0.3037992 | 3.793706 |
| r | 2616.00 | -0.1423474E-04 | -0.4285827E-03 |
| $r_w$ | 0.11 | 1573.013 | 1.991463 |
| $T_l$ | 113.00 | 0.3315279E-02 | 0.4311673E-02 |
| $H_l$ | 799.00 | -0.3037993 | -2.793706 |
| $K_w$ | 11470.00 | 0.7542402E-02 | 0.9956813 |
| L | 1429.00 | -0.6053978E-01 | -0.9956813 |
| 9 | R=Q = 46.98318 | $H_{wl}$ = 769.2566 | $H_{wu}$ = 1008.999 |
| $T_u$ | 74270.00 | 0.31442505E-08 | 0.4970363E-05 |
| $H_u$ | 1009.00 | 0.1949509 | 4.186722 |
| r | 14690.00 | -0.1402964E-05 | -0.4386581E-03 |
| $r_w$ | 0.10 | 934.7742 | 1.989593 |
| $T_l$ | 70.80 | 0.3460010E-02 | 0.5213967E-02 |
| $H_l$ | 768.00 | -0.1949509 | -3.186722 |
| $K_w$ | 10430.00 | 0.4481110E-02 | 0.9947811 |
| L | 1672.00 | -0.2795333E-02 | -0.9947811 |
| 10 | R=Q = 52.62038 | $H_{wl}$ = 739.0709 | $H_{wu}$ = 1024.999 |
| $T_u$ | 115300.00 | 0.1338124E-08 | 0.2932052E-05 |
| $H_u$ | 1025.00 | 0.1833462 | 3.571428 |
| r | 8017.00 | -0.2115667E-05 | -0.3223333E-03 |
| $r_w$ | 0.07457 | 1406.067 | 1.992582 |
| $T_l$ | 90.60 | 0.2167197E-02 | 0.3731408E-02 |
| $H_l$ | 738.00 | -0.1833462 | -2.571428 |
| $K_w$ | 12020.00 | 0.4361387E-02 | 0.9962656 |
| L | 1141.00 | -0.4594555E-01 | -0.9962656 |

## APPENDIX D - RANGE OF DETERMINISTIC NORMALIZED SENSITIVITY COEFFICIENTS

Table D-1 gives the deterministic normalized sensitivity coefficients for the extremes of the range of the independent parameters as shown in Table 2-1.

Table D-1. Normalized Sensitivity Coefficients for the Performance Measure Q.

| Input Parameter | Minimum of Range | Maximum of Range |
|---|---|---|
| $r_w$ | 1.999 | 1.982 |
| $r$ | $-0.174 \times 10^{-7}$ | $-0.690 \times 10^{-3}$ |
| $T_u$ | $0.213 \times 10^{-9}$ | $0.880 \times 10^{-5}$ |
| $H_u$-$H_l$ | 1.0 | 1.0 |
| $T_l$ | $0.213 \times 10^{-3}$ | $0.877 \times 10^{-2}$ |
| L | -1.0 | -0.991 |
| $K_w$ | 1.0 | 0.991 |

For the key parameters ($r_w$, $H_u$-$H_l$, L, and $K_w$), it is seen that these coefficients differ little at the extremes of the ranges studied. The other parameters remain unimportant over their respective ranges.

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